ATUL VIDYALAYA
SECOND PRELIMINARY EXAMINATION-2012-13
MATHEMATICS

STD:XII Science   **MM: 100**
DATE: __/__/____   **TIME: 3 hrs**

GENERAL INSTRUCTION
(Three hours)
*(Candidate are allowed additional 15 minutes for **only** reading the paper .*
*They must **NOT** start writing during this time)*

*There will be one paper of **three** hours duration of 100 marks. The syllabus is divided into three sections A , B and C. Section A is compulsory for all candidates. Candidates will have choice of attempting questions from **either** from Section B or Section C.*

*Section A(**80 marks**) will consists of 9 questions. Candidate will be required to answer **Question -1 (Compulsory)** and **five** out of the rest of the eight question.*

*Section B/C(**20 marks**) Candidate will be required to answer two questions out of three from either Section B or Section C.*

Section – A
Question 1.

i) Find the value of x , if $\sin^{-1}\left(\dfrac{5}{13}\right)+\sin^{-1}\left(\dfrac{7}{25}\right)=\cot^{-1}x$.

[3]

ii) Show that the equation $y=be^{x}+ce^{2x}$ is a solution of the differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0$$

[3]

iii) Find the area between the curve $x=y^{2}-2y$, the y – axis and the abscissa , $y=1$

and $y=2$.   [3]

iv) Evaluate : $\displaystyle\int_{0}^{3}\frac{x}{1+x^{2}}\,dx$

[3]

v) Find the modulus and amplitude of $\dfrac{2+3i}{3+2i}$.   [3]

vi) Find " c " of the Lagrange's Mean Value Theorem when $f(x)=x(x-2)$ in [1 , 2].

[3]

vii) Find the probability of getting an even number on the first die or a total of 8 in a

single throw of two dice .   [3]

iix) For the matrix $A=\begin{bmatrix}3 & 1\\ 7 & 5\end{bmatrix}$, find x and y so that $A^{2}=xI=yA$. Hence , find A^{-1} .[3]

ix) Simplify the Boolean expression : $x+x.y'+x'.y$

[3]

x) Evaluate : $\displaystyle\lim_{x\to\frac{\pi}{2}}(\sec x-\tan x)$

[3]

Question 2.

i) Prove that $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}=a^{3}+b^{3}+c^{3}-3abc$.   [5]

$$A = \begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix}$$

ii) If , find A^{-1}. Using A^{-1}, solve the system of linear equations :

$$4x - 5y - 11z = 12$$
$$x - 3y + z = 1$$
$$2x + 3y - 7z = 2$$

[5]

Question 3

i) Evaluate : $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ [5]

ii) Find the co-ordinate of the points on the ellipse $4x^2 + y^2 = 8$ at which tangents are parallel to the straight line $2x - y = 7$.   [5]

Question 4.

i) Suppose A and B are two equally strong table tennis players. Which of the following two events is more probable ?

a) A beats b in exactly 3 games out of 4 , or

b) A beats B in exactly 5 games out of 8 ?   [5]

ii) Find the probability of throwing 11 with three dice .   [5]
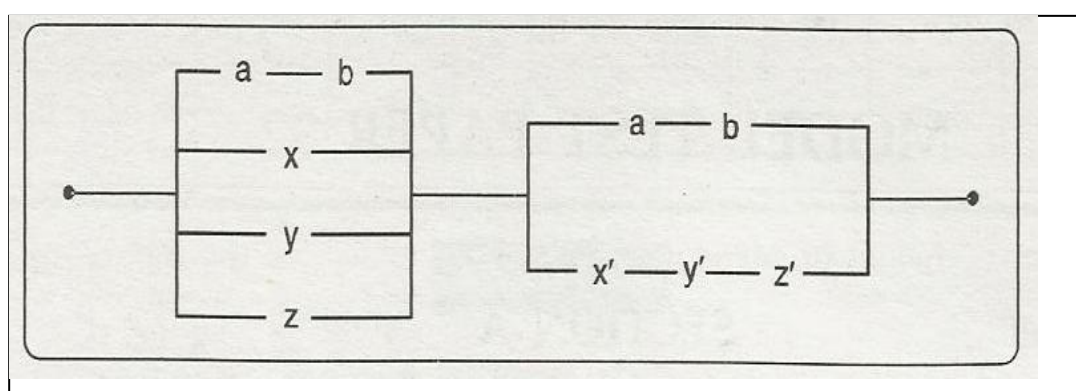
Question 5.

i) Evaluate : $\displaystyle\int x^2 \sin^{-1} x\, dx$ [5]

ii) Prove that the triangle of maximum area inscribed in a given circle must be equilateral.   [5]

Question 6.

i) Find the area of the region lying in the first quadrant bounded by the parabola $y^2 = 4x$, the x-axis and the ordinate $x = 4$.   [5]

ii) Verify Lagrange's Mean Value theorem for the given function in the given interval and find the value of " c " of this theorem $f(x) = \sqrt{x^2 - 1}$ in [1 , 3].   [5]

Question 7.

i) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $\dfrac{dy}{dx} = -\dfrac{1}{(1+x)^2}$.   [5]

ii) Write the Boolean expression for the following switching circuit :

Using laws of Boolean algebra , simplify the circuit and construct an equivalent

switching circuit . [5]

Question 8.

i) Calculate the coefficient of correlation between x and y for the following data : [5]

x	10	7	12	9	16	18	8	14
y	6	4	7	10	7	15	5	11

ii) If two coefficient of regression are – 0.6 and -1.4 , find the coefficient of correlation .[5]

Question 9

i) If 1 , ω , ω² are the three cube roots of unity , show that

$$x^3 - y^3 = (x-y)(\omega x - \omega^2 y)(\omega^2 x - \omega y)$$

[5]

ii) Solve : $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$

[5]

(Answer *two* questions from *either* Section B or Section C)

SECTION B

Question 10

i) A variable plane is at a constant distance p from origin and meets the axes in A , B and C . Through A , B , C planes are drawn parallel to the co-ordinate planes , show that the locus of their point of intersection is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$. [5]

ii) Find the equation to the line that intersects the lines $2x + y - 1 = 0, x + 2y + 3z = 0$ and

$3x - y + z + 2 = 0, 4x + 5y - 2z - 3 = 0$ and parallel to the line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$. [5]

Question 11

i) Prove that the triangle whose vertices have the position vectors

$2\hat{i} + 4\hat{j} - \hat{k}, 4\hat{i} + 5\hat{j} + \hat{k}, 3\hat{i} + 6\hat{j} - 3\hat{k}$ is an isosceles right angled triangle . [5]

ii) If $a = 4\hat{i} + 3\hat{j} - \hat{k}, b = 2\hat{i} + \hat{j} + 3\hat{k}$ and $c = \hat{i} - \hat{j} + 2\hat{k}$, then simplify $(a-b)X(b+c)$. [5]

Question 12

i) Urn – 1 contains 5 red and 5 black balls , urn – 2 contains 4 red and 8 black balls and urn -3 contains 3 red and 6 black balls. One urn is chosen at random and a ball is drawn . The colour of the ball is black . What is the probability that this has been drawn from urn – 3 ? [5]

ii) Mean and standard deviation of a binomial distribution are respectively 4 and $\sqrt{\dfrac{8}{3}}$. Find the value of n and p . [5]

SECTION C

Question 13

i) From the following data , giving the prices in arbitrary units and quantities of commodities A , B and C compute the weighted aggregate index of 1992 using 1986 as base year : [5]

Commodity	Quantities	Prices	
		1986	1992

A	7	321	581
B	8	54	67
C	4	224	305

ii) The following mid-day temperature in degree centigrades were recorded at a place In the U.K. for the first 14 days of January 1975 :

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Tem.	0	4	2	6	2	-1	-3	1	5	0	4	5	6	7

Calculate 5 day moving averages for this period and display them and the original table on the same graph .   [5]

Question 14

i) What is the actual rate of interest which a banker gets for the money when he discounts a bill legally due in 6 months at 5% p.a. ?   [5]

ii) A manufacturer produces nuts and bolts for industrial machinery . It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts , while it takes 3 hours on machine A and 1 nour on machine B to produce a package of bolts .He earns a profit of ` 2.50 per package on nuts and ` 1 per package on bolts . Form a linear programming problem to maximize his profit , if he operates each machine for at least 12 hours .   [5]

Question 15

i) What sum should be invested every year at 5% per annum compound interest for 20 years to replace plant and machinery which is expected to cost then , 25% more , over its present cost of ` 60 , 000 ?   [5]

ii) Given the cost function $C(x) = 50 + 2x$ and demand function $p = 100 - 4x$ .Find the

(a) equilibrium output

(b) Maximum price   [5]